A Course for Advanced Quantum Field Theory

Gauge Field Theories [§]

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Contents

1. Why Gauge Field Theories [1+4hr]

1.1. Why Quantum Field Theories: A Modern View

— Overview: Science, Physics and QFT

— A Brief History about QFT

— QFT: A Path Integral Summary

— Effective Field Theory as the Foundation of QFT

1.2. Why Gauge Field Theories

— All Fundamental Forces in Nature

— Gauge Revolution

1.3. Vacuum Energy, Cosmological Constant and Dark Energy

— Vacuum Energy from Quantum Field Theory

— Einstein Equation and Effective Cosmological Constant

— Friedman Equation and Connection to Dark Energy

1.4. A Condensed Matter Application: Order Parameter

— Order Parameter, Phase Transition, Spontaneous Symmetry Breaking

— Landau-Ginzburg Theory, Universality vs. Renormalization Group

2. Symmetries and Conservation Laws [3hr]

2.1. Symmetries and Currents

— Action Principle

— Noether Theorem

— Energy-Momentum Tensor

[§]For spring 2009. Updated on February 20, 2009.

2.2. Lorentz and Poincare Symmetries

— Lorentz Group and Irreducible Representations for Quantum Fields

— Poincare Group and its Casimir Operators

— Massless Particles and Helicity

— Master Group, No-Go Theorem and Supersymmetry

2.3. Weyl, Majorana and Dirac Fermions

— Definitions, Mass terms and Relations

— Representation of SM Quarks, Leptons and Neutrinos

— Majorana Neutrino and Seesaw Mechanism

2.4. Brief Review of Lie Groups *

3. Symmetries and Their Breaking [8-9hr]

3.1. Global and Local Symmetries

— Concept of Local Gauge Symmetry

— Abel Gauge Symmetry: Maxwell Theory and QED

— Non-Abel Gauge Symmetry: Gauge Sector and Fermion Sector

3.2. Gauge Invariance and Geometry

— Gauge Field as a Phase Field

— Gauge Field and Parallel Transport

— Gauge Field Strength and Curvature Tensor

— Nonintegrable Phase Factor and Global Formulation of Gauge Fields

3.3. Gravity as a Gauge Theory

— Equivalence Principle

— Principle of General Covariance vs. Gauge Invariance

— Recipe for Constructing Generally Covariant Action

Example-1: Scalar Fields in Curved Spacetime

Example-2: Gauge Fields in Curved Spacetime

Example-3: Einstein-Hilbert Action of Graviton Fields

3.4. Spontaneous Global Symmetry Breaking

— Explicit Symmetry Breaking vs. Spontaneous Symmetry Breaking (SSB)

— Physical Vacuum vs. Spontaneous Symmetry Breaking

— Goldstone Theorem and Three Ways of Proving It

— An Explicit Model of SSB and Nambu-Goldstone Bosons

— Pseudo-Nambu-Goldstone Bosons

— Goldstone Theorem vs. Spacetime Dimensions

3.5. Spontaneous Gauge Symmetry Breaking

— Anderson-Higgs Mechanism

Explicit Example-1: An Abel Higgs Model

Explicit Example-2: An Non-Abel Higgs Model

The Formal Proof

— Vacuum Energy from Spontaneous Symmetry Breaking

3.6. Superconductivity as a Higgs Phenomenon

— Bose-Einstein Condensation and Cooper Pair

— Photon Mass, London Penetration Depth, and Meissner Effect

— Flux Quantization and Abel Goldstone Boson

4. Path Integral Quantization: Gauge Fields [6-8hr]

4.1. Faddeev-Popov Quantization Method

— Redundant Gauge Orbits and Consistent Gauge Fixing

— Faddeev-Popov Determinant and Introduction of Ghosts

— Explicit Examples of Faddeev-Popov Quantization and Feynman Rules in Lorentz Gauge, Axial Gauge and R_ξ Gauge

— Gribov Copies

Example of Coulomb Gauge

Nonzero Topological Charge and Universality of Gribov Copies

4.2. BRST Symmetry and BRST Quantization

— BRST Transformations and Nilpotency

— Proof of BRST Symmetry for Quantized Lagrangian

4.3. Ward-Takahashi and Slavnov-Taylor Identities

— Ward-Takahashi Identities from BRST: A Review for QED

— Slavnov-Taylor Identities from BRST: General Gauge Theories

5. Renormalization of Gauge Theories [11-13hr]

5.1. Renormalization Program

— Renormalization Procedures

— Power Counting and Superficial Divergences

— Renormalization Counter Terms

— Multiplicative Renormalization vs. BPHZ Renormalization

5.2. Renormalization Types and Regularization Schemes

— Renormalization Types:

Non-renormalizable Theories (Effective Theories)

Renormalizable, Super-renormalizable and Finite Theories

— Regularization Schemes:
Dimensional Regularization
Pauli-Villars Regularization
Lattice Regularization

5.3. Renormalizability and Gauge Invariance

— Lee-Zinn-Justin Equations

— Proof of Renormalizability by Induction:
Pure Gauge Theories
Gauge Theories with Spontaneous Symmetry Breaking

— Proving Gauge Invariance of S-Matrix

5.4. Renormalization Group (RG)

— Concept of Renormalization Group

— Renormalization Group Equation

— Running Coupling Constant

— Ultraviolet and Infrared Fixed Points

— Callan-Symanzik Equation

— Weinberg Theorem and Asymptotic Solution of RG Equation

— Minimal Subtraction Scheme

— Bjorken Scaling and Scaling Violation

— Altarelli-Parisi Equation *

5.5. Renormalization of Non-Abelian Gauge Theory at One-Loop

— Lagrangian and Counter Terms

— Gauge Boson Self-Energy at One-Loop

— Fermion Self-Energy at One-Loop

— Vertex Corrections at One-Loop

5.6. Asymptotic Freedom of Non-Abelian Gauge Theory

— Computation of β-Function at One-Loop

— Asymptotic Freedom of QCD: Physical Interpretation

— Asymptotic Freedom and Spontaneous Symmetry Breaking

— Beta Function and QCD Scale Parameter

5.7. Background Field Method and Application to β-Function

— Background Field Method

— One-Loop Correction to the Effective Action

— Computation of Functional Determinants

6. Anomalies [5-6hr]

6.1. Chiral Anomalies: ABJ Anomaly and Non-Abelian Generalization

— Concept of Chiral Anomaly: Global vs. Gauge Anomalies

— Abelian Chiral Anomaly

— Extension to Non-Abelian Chiral Anomaly

— General Properties of Chiral Anomaly

— Application to $\pi^0 \to 2\gamma$

6.2. Path Integral Formulation of Chiral Anomalies

— Chiral Symmetry and Path Integral Measure

— Chiral Anomaly from Regularizing Jacobian of Quantum Measure

— Chiral Anomaly and Atiyah-Singer Index Theorem

6.3. Gauge Anomaly Cancellation Condition

6.4. Scale Anomaly

7. Electroweak Standard Model and Beyond [10-12hr]

7.1. Structure of the Standard Model

— Overview

— Gauge Anomaly Cancellation

7.2. The Standard Model Lagrangian

— Gauge-Higgs Sector and Weak Gauge Boson Masses

— Fermion-Gauge Sector: Charged and Neutral Currents

— Fermion-Higgs Yukawa Sector and Fermion Mass Generation

— Quark Mixing in Charged Currents: CKM Matrix
CKM Parametrization
Wolfenstein Parametrization
Jarlskog Invariant: Measure of CP-Violation
Unitarity Triangle

— Neutrino/Lepton Mixing in Charged Currents: MNSP Matrix
Neutrino Masses from Weinberg dim-5 Operator
Neutrino/Lepton Mixing and MNSP Matrix

— Neutrino Masses, Neutrino Oscillations and Neutrinoless Double-β Decay

7.3. R_ξ Gauge Quantization and Feynman Rules

— R_ξ Gauge-Fixing, Propagators and Feynman Rules

— SM Gauge Transformations and Ghost Lagrangian

— SM Feynman Rules in R_ξ Gauge

7.4. Higgs Mechanism and Equivalence Theorem

— Equivalence Theorem: Mathematical Formulation

— Equivalence Theorem as a Formulation of Higgs Mechanism

— Kaluza-Klein Equivalence Theorem and Geometric Higgs Mechanism
  Kaluza-Klein Compactification in 5d
  Geometric Higgs Mechanism and KK Equivalence Theorem

7.5. WW Scattering and Unitarity Bound

— WW Scattering and Probing the Electroweak Symmetry Breaking

— Partial Wave Analysis and Unitarity Condition

— Unitary Limits on Higgs Mass and on Scales of New Physics
  Unitary Limit on Higgs Mass
  Unitary Limit on the Scale of Electroweak Symmetry Breaking
  Unitary Limit on the Scales of Mass Generations for Quarks, Leptons and Neutrinos

7.6. Radiative Corrections

— Higgs Mass, Radiative Corrections and Fine-Tuning Problem

— Oblique Corrections and Screening Theorem
  Oblique Corrections at Z-Pole and W-Pole
  WT Identities and their Approximation in Gaugeless Limit
  One-Loop Oblique Corrections and Screening Theorem

— Coleman-Weinberg Potential *

7.7. Electroweak Chiral Lagrangian *

7.8. Dynamical Electroweak Symmetry Breaking *

8. Supersymmetry (SUSY) † [6-8hr]

8.1. SUSY and SUSY Algebra

— What is SUSY?

— SUSY Algebra

— Vacuum Energy and SUSY

— Structure of the Supersymmetric SM

— Wess-Zumino Model

8.2. Supersymmetric Lagrangian

— SUSY Lagrangian for Chiral Supermultiplets

— SUSY Lagrangian for Gauge Supermultiplets

— Summary: SUSY Interactions

8.3. Superspace and Superfields

— Review of Dotted and Undotted Indices of Weyl Spinors

— Superspace and Superfields

8.4. Soft SUSY Breaking

8.5. Minimal Supersymmetric SM (MSSM)

— MSSM Superpotential with and without R-Parity

— MSSM Higgs Sector: Higgs Masses and Little Fine-Tuning Problem

8.6. Gauge Unification

— Gauge Unification Problem in the SM

— Gauge Unification in Multi-Higgs Extension of the SM

— Gauge Unification in the MSSM

8.7. MSSM Spectrum, R-Parity and SUSY Dark Matter

8.8. Mechanism of Supersymmetry Breaking

9. Nonperturbative Aspects of Gauge Theories † [5-7hr]

9.1. Aharonov-Bohm Effect and Berry Phase

9.2. Vortices and Monopoles

— Vortices

— Dirac Monopoles, 't Hooft-Polyakov Monopoles, BPS Monopoles

9.3. Instanton, Vacuum Tunnelling and Strong CP Problem

9.4. Large-N Expansion

9.5. Lattice Gauge Theory

10. Grand Unification (GUT) † [4-6hr]

10.1. $SU(5)$ GUT

10.2. Coupling Constant Unification

10.3. Proton Decay

10.4. $SO(10)$ GUT *

10.5. Seesaw Mechanism and Cosmic Baryon Asymmetry

11. Perturbative Gravity † [3-4hr]

11.1. Einstein Gravity as an Effective Field Theory

11.2. Gravity Coupled to the Standard Model

11.3. Kaluza-Klein Compactification of 5D

11.4. Inflation, Cosmic Coincidence Problem

12. Field Theory and Collective Phenomena † [3hr]

12.1. Field Theory at Finite Temperature

12.2. Critical Phenomena and Mean Field Approximation

12.3. Quantum Hall Effect

12.4. Fractional Statistics, Chern-Simmons, and Topological Field Theory *

12.5. DNA and Quantum Field Theory *

Note:

— The topics marked with * will be taken as after-class reading under my guidance. Chapters marked by † will be moved to Tsinghua's fall semester class "Advanced Topics in Particle Theory" — I will continue to teach them for the last 1/3 of this course (about 20hrs).

— The first week of May is holiday, so the actual time for the course is $16 - 1 = 15$ weeks.

— Final Exam is in the 17th week.

— Future update of this Outline will appear on Web Site under "Gauge Field Theories":
http://hep.tsinghua.edu.cn/training/index.html

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