

A Course for Advanced Quantum Field Theory

Gauge Field Theories[§]

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Contents

1. Why Gauge Field Theories [1+4hr]

1.1. Why Quantum Field Theories: A Modern View

- A Brief History about QFT
- Effective Field Theory as the Foundation of QFT

1.2. Why Gauge Field Theories

- All Fundamental Forces in Nature
- Gauge Revolution

1.3. Vacuum Energy, Cosmological Constant and Dark Energy

- Vacuum Energy from Quantum Field Theory
- Einstein Equation and Effective Cosmological Constant
- Friedman Equation and Connection to Dark Energy

1.4. A Condensed Matter Application: Order Parameter

- Order Parameter, Phase Transition, Spontaneous Symmetry Breaking
- Landau-Ginzburg Theory, Universality vs. Renormalization Group

2. Symmetries and Conservation Laws [3hr]

2.1. Symmetries and Currents

- Action Principle
- Noether Theorem
- Energy-Momentum Tensor

2.2. Lorentz and Poincare Symmetries

- Lorentz Group and Irreducible Representations for Quantum Fields
- Poincare Group and its Casimir Operators
- Massless Particles and Helicity

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- Master Group, No-Go Theorem and Supersymmetry

2.3. Weyl, Majorana and Dirac Fermions

- Definitions, Mass terms and Relations
- Majorana Neutrino and Seesaw Mechanism

2.4. Brief Review of Lie Groups *

3. Symmetries and Their Breaking [8-9hr]

3.1. Global and Local Symmetries

- Concept of Local Gauge Symmetry
- Abel Gauge Symmetry: Maxwell Theory and QED
- Non-Abel Gauge Symmetry: Gauge Sector and Fermion Sector

3.2. Gauge Invariance and Geometry

- Gauge Field as a Phase Field
- Gauge Field and Parallel Transport
- Gauge Field Strength and Curvature Tensor
- Nonintegrable Phase Factor and Global Formulation of Gauge Fields

3.3. Gravity as a Gauge Theory

- Equivalence Principle
- Principle of General Covariance vs. Gauge Invariance
- Recipe for Constructing Generally Covariant Action
 - Example-1: Scalar Fields in Curved Spacetime
 - Example-2: Gauge Fields in Curved Spacetime
 - Example-3: Einstein-Hilbert Action of Graviton Fields

3.4. Spontaneous Global Symmetry Breaking

- Explicit Symmetry Breaking vs. Spontaneous Symmetry Breaking (SSB)
- Physical Vacuum vs. Spontaneous Symmetry Breaking
- Goldstone Theorem and Three Ways of Proving It
- An Explicit Model of SSB and Nambu-Goldstone Bosons
- Pseudo-Nambu-Goldstone Bosons
- Goldstone Theorem vs. Spacetime Dimensions

3.5. Spontaneous Gauge Symmetry Breaking

- Anderson-Higgs Mechanism
 - Explicit Example-1: An Abel Higgs Model
 - Explicit Example-1: An Non-Abel Higgs Model
 - The Formal Proof
- Vacuum Energy from Spontaneous Symmetry Breaking

3.6. Superconductivity as a Higgs Phenomenon

- Bose-Einstein Condensation and Cooper Pair
- Photon Mass, London Penetration Depth, and Meissner Effect
- Flux Quantization and Abel Goldstone Boson

4. Path Integral Quantization: Gauge Fields [6-8hr]

4.1. Faddeev-Popov Quantization Method

- Redundant Gauge Orbits and Consistent Gauge Fixing
- Faddeev-Popov Determinant and Introduction of Ghosts
- Explicit Examples of Faddeev-Popov Quantization and Feynman Rules in Lorentz Gauge, Axial Gauge and R_ξ Gauge
- Gribov Copies
 - Example of Coulomb Gauge
 - Nonzero Topological Charge and Universality of Gribov Copies

4.2. BRST Symmetry and BRST Quantization

- BRST Transformations and Nilpotency
- Proof of BRST Symmetry for Quantized Lagrangian

4.3. Ward-Takahashi and Slavnov-Taylor Identities

- Ward-Takahashi Identities from BRST: A Review for QED
- Slavnov-Taylor Identities from BRST: General Gauge Theories

5. Renormalization of Gauge Theories [11-13hr]

5.1. Renormalization Program

- Renormalization Procedures
- Power Counting and Superficial Divergences
- Renormalization Counter Terms
- Multiplicative Renormalization vs. BPHZ Renormalization

5.2. Renormalization Types and Regularization Schemes

- Renormalization Types:
 - Non-renormalizable Theories (Effective Theories)
 - Renormalizable, Super-renormalizable and Finite Theories
- Regularization Schemes:
 - Dimensional Regularization
 - Pauli-Villars Regularization
 - Lattice Regularization

5.3. Renormalizability and Gauge Invariance

- Lee-Zinn-Justin Equations
- Proof of Renormalizability by Induction:
 - Pure Gauge Theories
 - Gauge Theories with Spontaneous Symmetry Breaking
- Proving Gauge Invariance of S-Matrix

5.4. Renormalization Group (RG)

- Concept of Renormalization Group
- Renormalization Group Equation
- Running Coupling Constant
- Ultraviolet and Infrared Fixed Points
- Callan-Symanzik Equation
- Weinberg Theorem and Asymptotic Solution of RG Equation
- Minimal Subtraction Scheme
- Bjorken Scaling and Scaling Violation
- Altarelli-Parisi Equation *

5.5. Renormalization of Non-Abelian Gauge Theory at One-Loop

- Lagrangian and Counter Terms
- Gauge Boson Self-Energy at One-Loop
- Fermion Self-Energy at One-Loop
- Vertex Corrections at One-Loop

5.6. Asymptotic Freedom of Non-Abelian Gauge Theory

- Computation of β -Function at One-Loop
- Asymptotic Freedom of QCD: Physical Interpretation
- Asymptotic Freedom and Spontaneous Symmetry Breaking

5.7. Background Field Method and Application to β -Function

- Background Field Method
- One-Loop Correction to the Effective Action
- Computation of Functional Determinants

6. Anomalies [5-6hr]

6.1. Chiral Anomalies: ABJ Anomaly and Non-Abelian Generalization

- Concept of Chiral Anomaly: Global vs. Gauge Anomalies
- Abelian Chiral Anomaly
- General Properties of Chiral Anomaly

- Non-Abelian Chiral Anomaly and Application to $\pi^0 \rightarrow 2\gamma$

6.2. Path Integral Formulation of Chiral Anomalies

- Chiral Symmetry and Path Integral Measure
- Chiral Anomaly from Regularizing Jacobian of Quantum Measure
- Chiral Anomaly and Atiyah-Singer Index Theorem

6.3. Gauge Anomaly Cancellation Condition

6.4. Scale Anomaly

7. Electroweak Standard Model and Beyond [10-12hr]

7.1. Structure of the Standard Model

- Overview
- Gauge Anomaly Cancellation

7.2. The Standard Model Lagrangian

- Gauge-Higgs Sector and Weak Gauge Boson Masses
- Fermion-Gauge Sector: Charged and Neutral Currents
- Fermion-Higgs Yukawa Sector and Fermion Mass Generation
- Quark Mixing in Charged Currents: CKM Matrix
 - CKM Parametrization
 - Wolfenstein Parametrization
 - Jarlskog Invariant: Measure of CP-Violation
 - Unitarity Triangle
- Neutrino/Lepton Mixing in Charged Currents: MNSP Matrix
 - Neutrino Masses from Weinberg dim-5 Operator
 - Neutrino/Lepton Mixing and MNSP Matrix
- Neutrino Masses, Neutrino Oscillations and Neutrinoless Double- β Decay

7.3. R_ξ Gauge Quantization and Feynman Rules

- R_ξ Gauge-Fixing, Propagators and Feynman Rules
- SM Gauge Transformations and Ghost Lagrangian
- SM Feynman Rules in R_ξ Gauge

7.4. Higgs Mechanism and Equivalence Theorem

- Equivalence Theorem: Mathematical Formulation
- Equivalence Theorem as a Formulation of Higgs Mechanism
- Kaluza-Klein Equivalence Theorem and Geometric Higgs Mechanism
 - Kaluza-Klein Compactification in 5d
 - Geometric Higgs Mechanism and KK Equivalence Theorem

7.5. WW Scattering and Unitarity Bound

- WW Scattering and Probing the Electroweak Symmetry Breaking
- Partial Wave Analysis and Unitarity Condition
- Unitary Limits on Higgs Mass and on Scales of New Physics
 - Unitary Limit on Higgs Mass
 - Unitary Limit on the Scale of Electroweak Symmetry Breaking
 - Unitary Limit on the Scales of Mass Generations for Quarks, Leptons and Neutrinos

7.6. Radiative Corrections

- Higgs Mass, Radiative Corrections and Fine-Tuning Problem
- Oblique Corrections and Screening Theorem
 - Oblique Corrections at Z -Pole and W -Pole
 - WT Identities and their Approximation in Gaugeless Limit
 - One-Loop Oblique Corrections and Screening Theorem
- Coleman-Weinberg Potential *

7.7. Electroweak Chiral Lagrangian *

8. Nonperturbative Aspects of Gauge Theories [5-7hr]

8.1. Aharonov-Bohm Effect and Berry Phase

8.2. Vortices and Monopoles

- Vortices
- Dirac Monopoles, 't Hooft-Polyakov Monopoles, BPS Monopoles

8.3. Instanton, Vacuum Tunnelling and Strong CP Problem

8.4. Large- N Expansion

8.5. Lattice Gauge Theory

9. Supersymmetry (SUSY)[†] [6-8hr]

9.1. SUSY and SUSY Algebra

- What is SUSY?
- SUSY Algebra
- Vacuum Energy and SUSY
- Structure of the Supersymmetric SM
- Wess-Zumino Model

9.2. Supersymmetric Lagrangian

- SUSY Lagrangian for Chiral Supermultiplets
- SUSY Lagrangian for Gauge Supermultiplets
- Summary: SUSY Interactions

9.3. Superspace and Superfields

- Review of Dotted and Undotted Indices of Weyl Spinors
- Superspace and Superfields

9.4. Soft SUSY Breaking

9.5. Minimal Supersymmetric SM (MSSM)

- MSSM Superpotential with and without R -Parity
- MSSM Higgs Sector: Higgs Masses and Little Fine-Tuning Problem

9.6. Gauge Unification

- Gauge Unification Problem in the SM
- Gauge Unification in Multi-Higgs Extension of the SM
- Gauge Unification in the MSSM

9.7. MSSM Spectrum, R -Parity and SUSY Dark Matter

9.8. Mechanism of Supersymmetry Breaking

10. Grand Unification (GUT)[†] [4-6hr]

10.1. $SU(5)$ GUT

10.2. Coupling Constant Unification

10.3. Proton Decay

10.4. $SO(10)$ GUT^{*}

10.5. Seesaw Mechanism and Cosmic Baryon Asymmetry

11. Perturbative Gravity[†] [3-4hr]

11.1. Einstein Gravity as an Effective Field Theory

11.2. Gravity Coupled to the Standard Model

11.3. Kaluza-Klein Compactification of 5D

11.4. Inflation, Cosmic Coincidence Problem

12. Field Theory and Collective Phenomena[†] [3hr]

12.1. Field Theory at Finite Temperature

12.2. Critical Phenomena and Mean Field Approximation

12.3. Quantum Hall Effect

12.4. Fractional Statistics, Chern-Simmons, and Topological Field Theory^{*}

12.5. DNA and Quantum Field Theory^{*}

Note:

- The topics marked with * will be taken as after-class reading under my guidance. Chapters marked by [†] will be moved to Tsinghua's fall semester class "Advanced Topics in Particle Theory" — I will continue to teach them for the last 1/3 of this course (about 20hrs).
- The first week of May is holiday, so the actual time for the course is $16 - 1 = 15$ weeks.
- Final Exam is in the 17th week.
- For future update of this List of Content, please check our Web site under "Gauge Field Theories":
<http://hep.tsinghua.edu.cn/training/index.html>

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Time and Location:

Every Monday and Wednesday evening (7:20pm-8:55pm).

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